LASER ACTION IN STELLAR ENVELOPES

II. He I

Y. P. VARSHNI and R. M. NASSER

Department of Physics, University of Ottawa, Canada K1N6N5

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Abstract. Model calculations for laser action in He I are carried out, when helium plasma is rapidly cooled by expansion. Results are presented for four transitions, two of which, $3^1S \rightarrow 2^1P^0(\lambda 7281)$ and $3^1D \rightarrow 2^1P^0(\lambda 6678)$, show strong population inversion. Available observational evidence for possible laser action in these two lines in Wolf–Rayet and emission-line stars is summarized and discussed.

1. Introduction

The suggestion of Gudzenko and Shelepin (1963, 1965) that if an ionized gas is rapidly cooled, population inversions will occur during the subsequent recombination cascade, has led to a lot of theoretical and experimental work. This scheme is capable of generating intense laser action and it is known as the plasma laser or recombination laser. Rapid cooling of a strongly ionized plasma leads to a rapid recombination. If the electron temperature is sufficiently low, the recombination occurs preferentially into the upper excited states of the ion, which then decay by radiative and/or collisional cascade to the ground state. Within the cascade from the upper excited states, population inversions may form among the excited states depending on the relative transition probabilities. The work of Gudzenko and Shelepin has been amplified and extended by Gudzenko *et al.* (1966, 1967). Reviews of the subject have been published by Gudzenko *et al.* (1974, 1975). A short review has been given by Otsuka (1980).

Several methods of cooling the plasma have been suggested. Two of these which are of importance to astrophysics are the following: (a) rapid expansion, and (b) contact with a neutral gas. Several types of stars (e.g., Wolf–Rayet, Of) are known to undergo mass loss, and if the mass loss is rapid enough, it can lead to rapid cooling. The contact with the interstellar gas can provide further cooling. In the astrophysical context, the term laser action actually means amplified spontaneous emission as there are no mirrors.

The properties of a recombining plasma are usually investigated in the framework of the Collisional-Radiative (CR) model proposed by Bates *et al.* (1962a, b). Hydrogen and hydrogen-like ions, being simple systems have been extensively investigated, both theoretically and experimentally. Theoretical studies have been carried out by Bates and Kingston (1963), Gordiets *et al.* (1968a, b), Bohn (1971, 1974, 1978), Fujimoto *et al.* (1972), Skorupski and Suckewer (1974a, b), Pert and Ramsden (1974), Reshetnyak and Shelepin (1975), Jones and Ali (1975, 1977, 1978), Ali and Jones (1975, 1976), Pert (1976, 1979), Luk'yanov (1977), Tallents (1977, 1978), Otsuka (1978), Furukane *et al.*

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(1979), Furukane and Yamamoto (1980), Pert and Tallents (1981), Furukane *et al.* (1982, 1983), Boiko *et al.* (1982), Oda and Furukane (1983), and Furukane and Oda (1984).

Experimental observations of population inversion in hydrogen have been made in hydrogen by Hoffmann and Bohn (1972), Suckewer *et al.* (1976), Lukyanov *et al.* (1978), Hara *et al.* (1980a, b), Trebes *et al.* (1981), and Miyake *et al.* (1983), in He II by Suckewer *et al.* (1976), Sato *et al.* (1977), and Hara *et al.* (1982), in C VI by Irons and Peacock (1974), Dewhurst *et al.* (1976), Dixon *et al.* (1978), Key *et al.* (1979), and Jacoby *et al.* (1981), and in O VIII by Korukhov *et al.* (1982).

Varshni and Lam (1976) presented calculations on population inversion and laser action in He II in the context of stellar envelopes. In the present paper we report model calculations for laser action in certain transitions of He I, using the CR model. First we summarize the work which has been done on population densities and population inversion in He I in laboratory plasma.

An early discussion of the possibility of producing population inversion in helium is due to Gordiets *et al.* (1968b). Theoretical calculations on population densities of He I levels under different conditions have been made by Johnson (1967), Drawin (1969), Brocklehurst (1972), Limbaugh and Mason (1971), Drawin *et al.* (1973), Filippov and Yevstigneyev (1975), Hess and Burrell (1979), Fujimoto (1979), Srivastava and Ghosh (1981), Ilmas and Nugis (1982), and Furukane and Oda (1985).

The experiments of Goldfarb *et al.* (1969) on axisymmetric supersonic jets of a helium plasma revealed a deviation from an equilibrium energy distribution of He atoms at the end of the nozzle of a plasmatron and showed that a population inversion of levels with principal quantum numbers $n = 4$ and 3 occurs at some finite distance from the nozzle.

Johnson (1967) and Johnson and Hinnov (1969) give the results of the measurement of population densities of excited states of neutral helium in afterglow discharges. The data show population inversion between certain levels with $n = 4$ and 3 and also between certain levels with $n = 5$ and 3.

Zhinzhikov *et al.* (1978) have studied the population inversion in He I levels with $n = 4$ and 3 in the supersonic expansion of helium plasma from circular and plane nozzles into a low density medium. In an axisymmetric jet the population inversion occurred at some finite distance from the nozzle; in plane jets the inversion occurred immediately beyond the slit nozzle. The experimental results were compared with theoretical calculations; the experimental results confirm the operation of a recombination mechanism.

Recently, Hara *et al.* (1985) have observed population inversions in a freely-expanding helium plasma. Experimental results showed that population inversions for $n = 3$–4 and 3–5 levels of He singlet system are larger than the triplet system.

Besides helium atom, population inversion has also been observed in some helium-like ions (Bhagavatula and Yaakobi, 1978; Zhizhan and Guangyu, 1983).

2. The Theoretical Model

We consider what happens when the plasma in the outer layers of a star rapidly expands. To focus our discussion we consider Wolf–Rayet stars. The atmospheres of these stars depart seriously from a condition of local thermodynamic equilibrium (LTE). We consider the state of plasma at the base of the extended atmosphere of a Wolf–Rayet star, roughly where its 'photosphere' would lie. We make the reasonable assumption that the conditions there will also correspond to non-LTE. Hence, to obtain the relative concentrations of He, He^+, and He^{++} at a particular electron density (n_e) and electron temperature (T_e) we use the non-LTE method of House (1964). It is well known that a high-speed mass loss occurs from Wolf–Rayet stars. We assume that this plasma expands adiabatically, for which the plasma density N and T_e are related by $NT_e^{1-\gamma} = \text{const}$. We assume $\gamma = \frac{5}{3}$; for the actual plasma the value will be slightly smaller. The flow is supersonic and to a first approximation, the plasma is assumed to be 'frozen' during the rapid fall of temperature, for which a factor of 5 is assumed. (The same factor for the fall in the temperature has been used by Gudzenko *et al.*, 1966; and Varshni and Lam, 1976.) Rapid recombinations will occur in this cooled plasma. The populations of the atomic levels can be calculated from the CR model. The time development of the population density n_i of a level i is described by the differential equation

$$\dot{n}_i \equiv \frac{\mathrm{d}n_i}{\mathrm{d}t} = -n_e n_i S_i - n_e n_i \sum_{j \neq i}^{\infty} C_{ij} - n_i \sum_{j=1}^{i-1} A_{ij} + \\ + n_e \sum_{j \neq 1}^{\infty} n_j C_{ji} + \sum_{j=i+1}^{\infty} n_j A_{ji} + \{\beta_i + \alpha_i n_e\} n_e n^+ , \tag{1}$$

where i and j symbolically represent energy states available to the bound electron. The meanings of other symbols in Equation (1) are as follows: n^+, ion density; n_e, electron density; C_{ij}, rate coefficient for excitation ($i < j$) or de-excitation ($i > j$) from level i to j by electronic collision; A_{ij}, Einstein coefficient for radiative transition from level i to j; S_i, rate coefficient for ionization from level i by electron collision; α_i, that for three-body recombination to level i; β_i, that for radiative recombination.

There is such an equation for each and every discrete level $i = 1, 2, 3, \ldots, \infty$. Thus, we obtain an infinite number of coupled differential equations describing the population densities of all the discrete levels. To solve it, the following assumptions are used. First, for all levels i located above a sufficiently high lying level r, the population density is assumed to be given by the Saha–Boltzmann equilibrium equation instead of by Equation (1). Thus, in this case, we have

$$n_i = n_i^E \equiv Z_i n_e n^+ \quad \text{for } i > r , \tag{2}$$

where

$$Z_i = \{g_i/2\omega^+\} (h^2/2\pi mkT_e)^{3/2} \exp(\chi_i/kT_e) , \tag{3}$$

where g_i and ω^+ are the statistical weight and the partition function of the ion, respectively, χ_i is the ionization potential of the level i, and the other symbols have the usual meaning. Second, the time derivative of n_i, Equation (1), is zero for all the levels $i \leq r$ except the ground state (1^1S). This leads to coupled linear equations for the levels $2^{1,3}S \leq i \leq r$ instead of the coupled differential equations. The upper limit in the summation in Equation (1), in actual practice is taken to be a high-lying level s, where $s > r$.

Equation (1) can be rewritten in term of the normalized population density which is defined by $\rho_i = n_i/n_i^E$.

$$\frac{\dot{n}_i}{n_i^E} = -\left(n_e S_i + n_e \sum_{j \neq i}^{s} C_{ij} + \sum_{j=1}^{i-1} A_{ij}\right)\rho_i + \sum_{j=i+1}^{r} \left\{C_{ij} n_e + \frac{Z_j}{Z_i} A_{ji}\right\} \rho_j +$$

$$+ \sum_{j>r}^{s} \left\{C_{ij} n_e + \frac{Z_j}{Z_i} A_{ji}\right\} + \frac{1}{Z_i} \{\alpha_i n_e + \beta_i\} = 0\,, \tag{4}$$

with $i = 2, 3, \ldots, r$.

The solution of $r - 1$ linear simultaneous equation, in terms of local ground state population density, may be written as

$$\rho_i = r_i^{(0)} + r_i^{(1)} \rho_1 \quad \text{for } 2 \leq i \leq r\,, \tag{5}$$

$r_i^{(0)}$ is the contribution from the continuum towards ρ_i, and $r_i^{(1)}$ is the contribution form the ground state towards ρ_i. Substituting Equation (5) into Equation (4), we have two set of $r - 1$ equations for $r_i^{(0)}$ and $r_i^{(1)}$ with $2 \leq i \leq r$. The solutions $r_i^{(0)}$ and $r_i^{(1)}$ obtained from these two sets of equations give the population density,

$$n_i = Z_i r_i^{(0)} n + n_e + \frac{Z_i}{Z_1} r_i^{(1)} n_1\,. \tag{6}$$

The atomic data used in the calculations are summarized in the next section.

3. Atomic Data

3.1. Energy Levels

The excitation energy of each level is based on Martin (1973). All of the levels having a principal quantum number $n \leq 8$ are treated as individual levels except for the levels having the orbital angular momentum $l \geq 4$, which for the same n, are grouped together to form a single level. For the levels with $n > 8$ all of the $S, P, D, \ldots$ levels are grouped together in one level, which was approximated by a hydrogenic level having a statistical weight twice that of hydrogen. The upper limit of the levels, r and s in CR model calculation are taken as $n_r = 15$ and $n_s = 22$, respectively. Therefore, we have 69 distinct levels in the main system, and the total number of levels whose population densities is calculated is 62. The energies and weight factors for these levels are given in Table I.

TABLE I

Energy levels of He I used in the model

Level No. (i)	State	Energy E_i (cm^{-1})	g_i	Level No. (i)	State	Energy E_i (cm^{-1})	g_i
1	1^1S	0.0	1	32	6^3D	195260.49	15
2	2^3S	159856.07	3	33	6^1D	195260.86	5
3	2^1S	166277.55	1	34	6^3F	195262.49	21
4	2^3P	169086.94	9	35	6^1F	195262.50	7
5	2^1P	171135.00	3	36	$6^{1,3}L$	195262.85	70
6	3^3S	183236.89	3	37	6^1P	195273.04	3
7	3^1S	184864.55	1	38	7^3S	195868.35	3
8	3^3P	185564.68	9	39	7^1S	195979.04	1
9	3^3D	186101.65	15	40	7^3P	196027.40	9
10	3^1D	186105.07	5	41	7^3D	196069.73	15
11	3^1P	186209.47	3	42	7^1D	196070.22	5
12	4^3S	190298.21	3	43	7^3F	196071.27	21
13	4^1S	190940.33	1	44	7^1F	196071.28	7
14	4^3P	191217.13	9	45	$7^{1,3}L$	196072	122
15	4^3D	191444.59	15	46	7^1P	196079.24	3
16	4^1D	191446.56	5	47	8^3S	196461.42	3
17	4^3F	191451.98	21	48	8^1S	196534.88	1
18	4^1F	191451.99	7	49	8^3P	196566.82	9
19	4^1P	191492.82	3	50	8^3D	196595.18	15
20	5^3S	193347.09	3	51	8^1D	196595.54	5
21	5^1S	193663.63	1	52	8^3F	196596.17	21
22	5^3P	193800.80	9	53	8^1F	196596.17	7
23	5^3D	193917.24	15	54	$8^{1,3}L$	196597	182
24	5^1D	193918.39	5	55	8^1P	196601.51	3
25	5^3F	193921.18	21	56	9	196955	324
26	5^1F	193921.19	7	57	10	197213	400
27	$5^{1,3}L$	193922	36	58	11	197403	484
28	5^1P	193942.57	3	59	12	197548	576
29	6^3S	194936.23	3	60	13	197661	676
30	6^1S	195115.00	1	61	14	197750	784
31	6^3P	195192.91	9	62	15	197822	900

3.2. Transition probabilities

Spontaneous transition probabilities were calculated using the formula

$$A_{nl,\,n'l'} = \frac{8\pi^2 e^2 g_{n'l'}}{mc^3 g_{nl}}\, \nu^2_{nl,\,n'l'} f_{n'l',\,nl}\,, \tag{7}$$

where $g_{n'l'}$ and g_{nl} are the statistical weights of the level $n'l'$ and nl, respectively, $\nu_{nl,\,n'l'}$ the frequency of the photon emitted as result of the transition and $f_{n'l',\,nl}$ is the absorption oscillator strength for the transition. When the angular momentum states l or l' are grouped together, then

$$A_{nl_i\ldots l_k,\,n'l'_i\ldots l'_k} = \frac{8\pi^2 e^2 \nu^2 \sum\limits_{l_i}^{l_k} \sum\limits_{l'_i}^{l'_k} g_{n'l'} f_{n'l',\,nl}}{mc^3 \sum\limits_{l_i}^{l_k} g_{nl}}\,. \tag{8}$$

Oscillator strengths for all the allowed transitions between the states with $n \leq 8$ and $l \leq 2$ were taken from Kono and Hattori (1984). These authors used variational wave function and they estimate the accuracy of their results to be better than 1% for most of the transitions and better than 0.1% for about a third of the transitions. For the allowed transitions between $n^{1,3}D$ and $n^{1,3}F$, we used the Coulomb approximation method (Bates and Damgaard, 1949) to calculate the oscillator strength. For all other transitions the hydrogenic formula was used to calculate the oscillator strength. For the optically forbidden transition $1^1S \rightarrow 2^3P$, the calculated transition probability by Garstang (1967) was adopted while experimental values are used for $1^1S \rightarrow 2^1S$ (van Dyck *et al.*, 1970) and for $1^1S \rightarrow 2^3S$ (Moos *et al.*, 1973). Other forbidden transitions were neglected.

3.3. Collisional excitation and dexcitation rate coefficients

The excitation rate coefficients for transition between bound levels were computed from the respective cross sections by integrating the cross section over a Maxwellian energy distribution. For most of the cross sections for low-lying levels the best available experimental and theoretical data were fitted to semi-empirical formulas. For all such transitions for which no experimental or theoretical data are available, extrapolated values of the parameters in the semi-empirical formula were used.

For allowed transitions between low-lying levels, the modified form of the Drawin (1963, 1964, 1967) formula proposed by Millette and Varshni (1980) was used:

$$\sigma(nl \rightarrow n'l') = 4\pi a_0^2 \alpha \frac{f_{nl,n'l'}}{E_{nl,n'l'}^2} \frac{U - \phi}{U^2} \ln(1.25\beta U), \tag{9}$$

where $E_{nl,n'l'}$ is the threshold energy for the excitation of the $nl \rightarrow n'l'$ transition in Rydberg, $U = E/E_{nl,n'l'}$ is the energy of the impacting electron E in threshold units, $f_{nl,n'l'}$ is the absorption oscillator strength, α, β, and ϕ are the fitting parameters which depend on the transition. In Equation (9), and the three subsequent equations, nl represents the lower level. This formula was fitted to the following cross-sections:

$1^1S \rightarrow 2^1P$ and $1^1S \rightarrow 3^1P$: Experimental cross-sections of Westerveld *et al.* (1979). $1^1S \rightarrow 4^1P$: Experimental cross-sections of Donaldson *et al.* (1972). $1^1S \rightarrow 5^1P$: Experimental cross-sections of Moustafa Moussa *et al.* (1969). $2^1S \rightarrow 2^1P$ and $2^3S \rightarrow 2^3P$: Theoretical cross-sections of Burk *et al.* (1969). $2^1S \rightarrow 3^1P$: Theoretical cross-sections of Flannery and McCann (1975). $2^1S \rightarrow 4^1P$: Theoretical cross-sections of Flannery *et al.* (1974). $2^3S \rightarrow 3^3P$, $2^3S \rightarrow 4^3P$, and $2^3S \rightarrow 5^3P$: Theoretical cross-sections of Ton-That *et al.* (1977).

For the remaining allowed transitions, the semi-classical impact parameter (IP) method (Burgess, 1964) was used for calculating the cross-sections of transitions with $\Delta n \leq 3$. For transitions with $\Delta n > 3$ we used the Van Regenmorter (1962) expression for calculating the cross-sections.

For optically forbidden transitions without change in spin, we used the modified form of Drawin, proposed by Millette and Varshni (1980):

$$\sigma(nl \to n'l') = 4\pi a_0^2 (n/n')^3 \frac{\alpha}{E^2_{nl,\,n'l'}} \frac{U - \phi}{U^2}\,, \tag{10}$$

where α and ϕ are parameters and the other symbols have their usual meaning. This formula was fitted to the following cross-sections:

$1^1S \to 2^1S$: Experimental cross-sections of Rice *et al.* (1972). $1^1S \to 3^1S$, $1^1S \to 3^1D$, $1^1S \to 4^1D$, and $1^1S \to 5^1D$: Experimental cross-sections of Moustafa Moussa *et al.* (1969). $1^1S \to 4^1S$: Experimental cross-sections of Van Rann *et al.* (1971). $1^1S \to 5^1S$: Experimental cross-sections of Pochat *et al.* (1972). $2^1S \to 3^1S$ and $2^1S \to 3^1D$: Theoretical cross-sections of Flannery and McCann (1975). $2^3S \to 3^3S$: Theoretical cross-sections of Khayrallah *et al.* (1978). $2^3S \to 3^3D$, $2^3S \to 4^3S$, $2^3S \to 4^3D$, $2^3S \to 4^3F$, $2^3S \to 5^3S$, $2^3S \to 5^3D$, and $2^3S \to 5^3F$: Theoretical cross-sections of Ton-That *et al.* (1977). $1^1S \to 4^1F$: Theoretical cross-sections of Van den Bos (1969). $1^1S \to 5^1F$: Theoretical cross-sections of Moiseiwitsch and Smith (1968).

For other optically forbidden transitions without change in spin between levels with $n \leq 8$ cross-sections were calculated using the classical binary encounter theory (Burgess, 1964; Vriens, 1966; Burgess and Percival, 1968).

For optically-forbidden transitions with change in spin, we used the following semi-empirical formula:

$$\sigma(nl \to n'l') = 4\pi a_0^2 (n/n')^3 \frac{1}{E^2_{nl,\,n'l'}} \left(\alpha \frac{U - \phi}{U^k} + \frac{\beta}{\sqrt{U}}\right), \tag{11}$$

where α, β, ϕ, and k are parameters and the other symbols have their usual meaning. $k = 5$ in all cases except for $2^1S \to 2^3P$, $2^3S \to 2^1P$, and $2^3S \to 2^1S$ transitions, for which k was taken equal to 2. This formula was fitted to the following cross-sections:

$1^1S \to 2^3P$: Experimental cross-sections of Jobe and St-John (1967). $1^1S \to 2^3S$: Theoretical cross-sections of Lin de Barros *et al.* (1975). $1^1S \to 3^3S$, $1^1S \to 4^3S$, $1^1S \to 5^3S$, $1^1S \to 3^3P$, $1^1S \to 4^3P$, $1^1S \to 5^3P$, $1^1S \to 3^3D$, and $1^1S \to 4^3D$: Experimental cross-sections of Moustafa Moussa *et al.* (1969). $2^3S \to 2^1P$, $2^3S \to 2^1S$, and $2^1S \to 2^3P$: Theoretical cross-sections of Burk *et al.* (1969).

For other forbidden transitions with change in spin between $n \leq 8$, we have used the expression given by Summers (1977) which was deduced from symmetrized classical binary encounter theory and corrected to the final states according to suggestion by Burgess (1964) and Burgess and Percival (1968). The dexcitation rate coefficients are obtained from the principle of detailed balance.

3.4. Ionization and three-body recombination rate coefficients

Experimental cross-section data are available only for 1^1S (Rapp *et al.*, 1965) and 2^3S (Dixon *et al.*, 1976) levels, and the theoretical cross-sections from 2^1S (Ton-That *et al.*, 1977). These were fitted to the semi-empirical formula (Drawin, 1967):

$$\sigma(nl \to c) = 2.66\pi a_0^2 \frac{\xi}{E^2_{nl}} \frac{U - 1}{U^2} \ln(1.25\beta U)\,, \tag{12}$$

The numerical value of ξ for ionization from the ground state is given by Drawin (1967), for $n > 1$, $\xi = 1$, and the other symbols have their usual meaning. For the resolved levels, the Drawin formula was used, and the hydrogenic approximation was used for unresolved levels. The ionization rate coefficients are calculated for a Maxwellian distribution of electrons. The three-body recombination rate coefficients are obtained from the principle of detailed balance.

3.5. Radiative recombination

The tables by Marr and West (1976) give the recommended experimental photoionization cross-sections from the ground state, and the theoretical cross-sections of Jacobs (1973) have been used for the levels 2^1S, 2^3S, 2^1P, and 2^3P. These cross-sections were fitted to a semi-empirical formula suggested by Millette and Varshni (1980): i.e.,

$$\sigma(U) = \frac{C}{U^p}\left[1 + \frac{a_1}{U} + \frac{a_2}{U^2} + \cdots + \frac{a_m}{U^m}\right], \tag{13}$$

where U is the energy of the incident photon in threshold unit s and $p, a_1, a_2, \ldots, a_m$ are parameters. Hydrogenic approximation was adopted to calculate the photoionization cross-sections for other levels. The radiative recombination rate coefficient was obtained by numerical integration.

4. Results and Discussion

The initial density of helium atoms before expansion was taken to be 1×10^{14} cm^{-3}. Calculations were carried out for the population densities of 62 levels of He I for a grid of n_e and T_e values.

Population inversion of varying degrees was found to occur for a great many transitions, including several between $n = 4$ and $n = 3$, for appropriate values of n_e and T_e. However, most of them either fall in the infrared region or else the population inversion is not very large. Two transitions which show large population inversion and which fall in the visible region are $3^1S \rightarrow 2^1P^0(\lambda 7281)$, and $3^1D \rightarrow 2^1P^0(\lambda 6678)$. To show the pattern of the results, we also include the next member of these two series, $4^1S \rightarrow 2^1P^0(\lambda 5047)$ and $4^1D \rightarrow 2^1P^0(\lambda 4922)$. We present the results for these four transitions. The population inversion is often measured in terms of P, where P is given by

$$P = N_k/g_k - N_i/g_i\,, \tag{14}$$

where N_i and N_k are the atomic populations of the lower and upper levels, and g_i and g_k are the respective statistical weights. P is sometimes called the 'overpopulation density' (Oda and Furukane, 1983). P is related to the fractional gain per unit distance α at the centre of a Doppler-broadened line by the following expression (cf. Willett, 1974)

$$\alpha = \left(\frac{\ln 2}{\pi}\right)^{1/2}\left[\frac{g_k A_{ki}}{4\pi}\right]\frac{P\lambda_0^2}{\Delta\nu}\,, \tag{15}$$

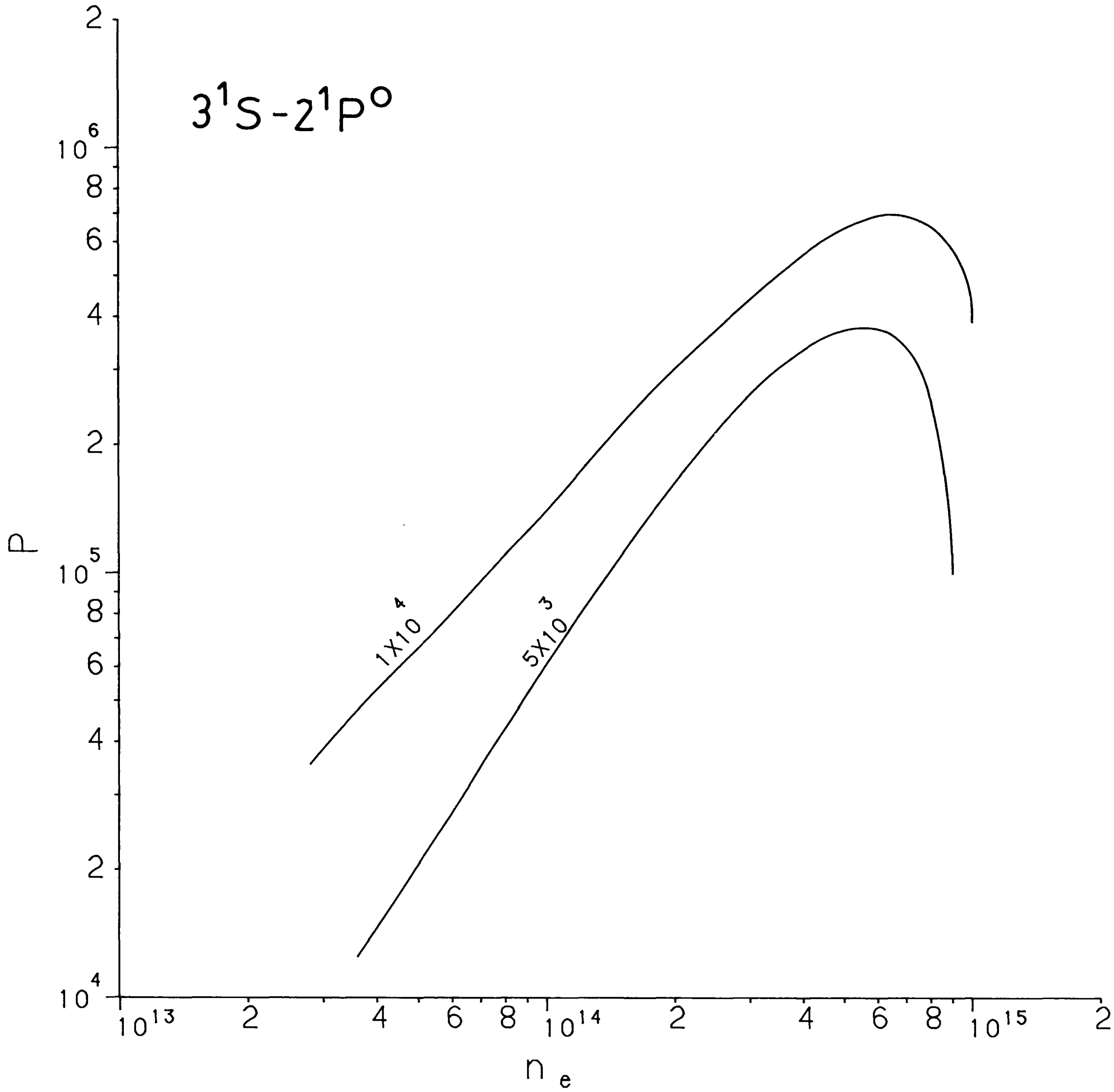

Fig. 1. P as a function of n_e for λ7281. The numbers by the side of curves represent T_e values in K.

where λ_0 is the centre wavelength of the transition, and $\Delta\nu$ is the linewidth. α describes the intensity of a plane wave at λ_0 according to

$$I = I_0 e^{\alpha l}, \tag{16}$$

where l is the length over which gain occurs.

In Figures 1 to 4 we show the variation of P with n_e for some typical values of T_e for the four transitions under consideration. From such plots, contours of equal P on a n_e, T_e diagram were made and the results are shown in Figures 5 to 8. It will be noticed that in all the four cases, strong population inversion occurs only in a small region. The magnitude of maximum population inversion for λ5048 and λ4922 is seen to be much smaller than that for λ7281 and λ6678. Thus the latter two lines are the most promising lines for undergoing laser action by the rapid cooling through expansion mechanism.

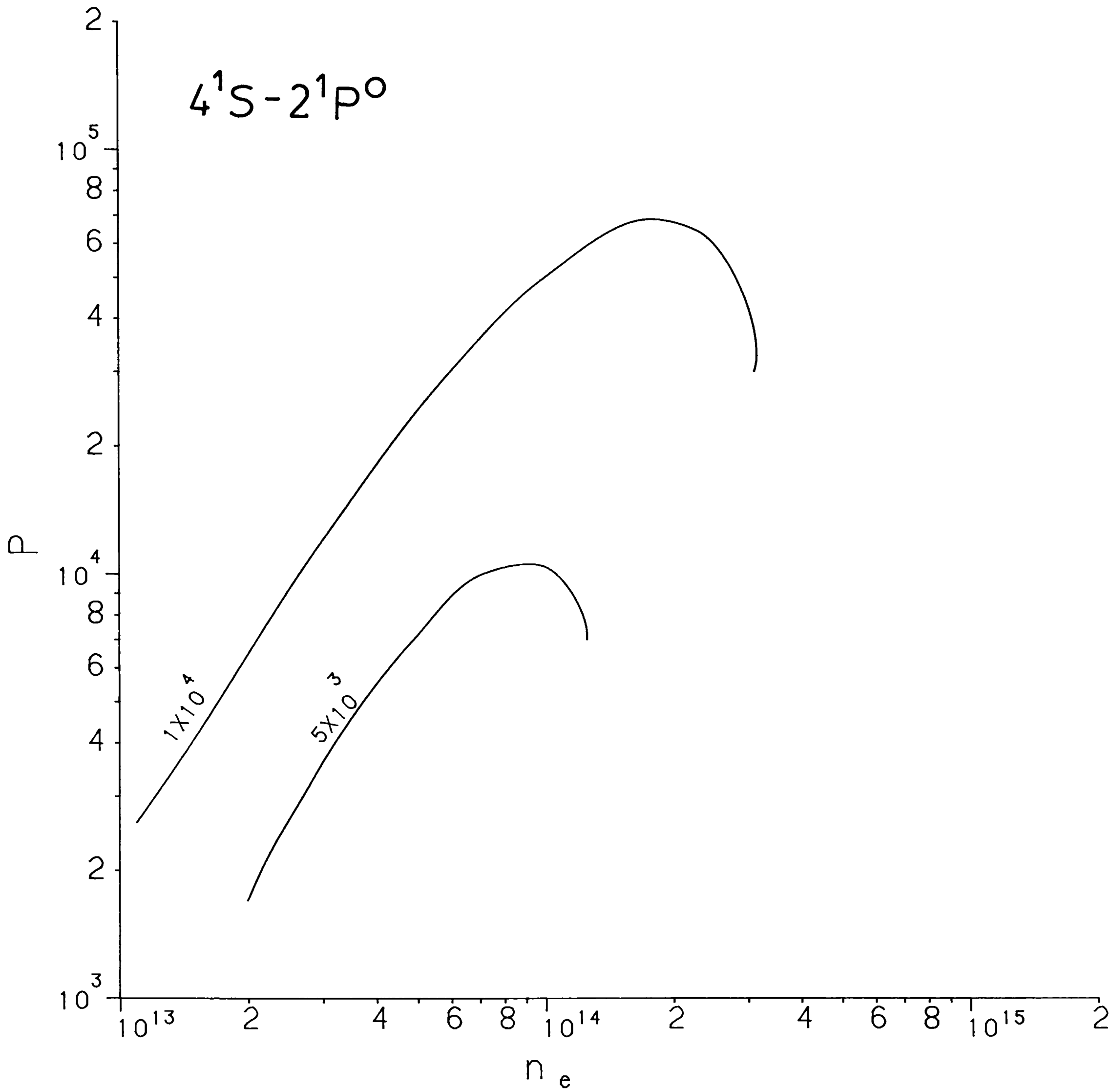

Fig. 2. P as a function of n_e for $\lambda 5048$. The numbers by the side of curves represent T_e values in K.

Further, it will be noticed (Figures 5 and 7) that there is a considerable overlap in the regions of maximum population inversion for the two lines. If we assume $\Delta\nu$ to be the same for both the transitions, the gain factor α is proportional to $g_k A_{ki} P \lambda_0^2$. For a point common to the maximum population inversion regions for the two lines, we have $P(6678) \simeq 3 \times 10^5 \text{ cm}^{-3}$ and $P(7281) \simeq 1 \times 10^6 \text{ cm}^{-3}$. With these values we find that $\alpha(6678)/\alpha(7281) \simeq 4.4$. Thus, though $P(6678)$ is less than $P(7281)$, the $\lambda 6678$ line would be much stronger than $\lambda 7281$. It is of some interest to note (Figures 6 and 8) that for $\lambda 5048$ and $\lambda 4921$ also there is considerable overlap in the regions of maximum population inversion. Here we wish to emphasize that the calculations that we have presented are for a simple model and strong laser action, in lines other than $\lambda 6678$ and $\lambda 7281$ is feasible through other mechanisms, for instance, by interaction with an ambient

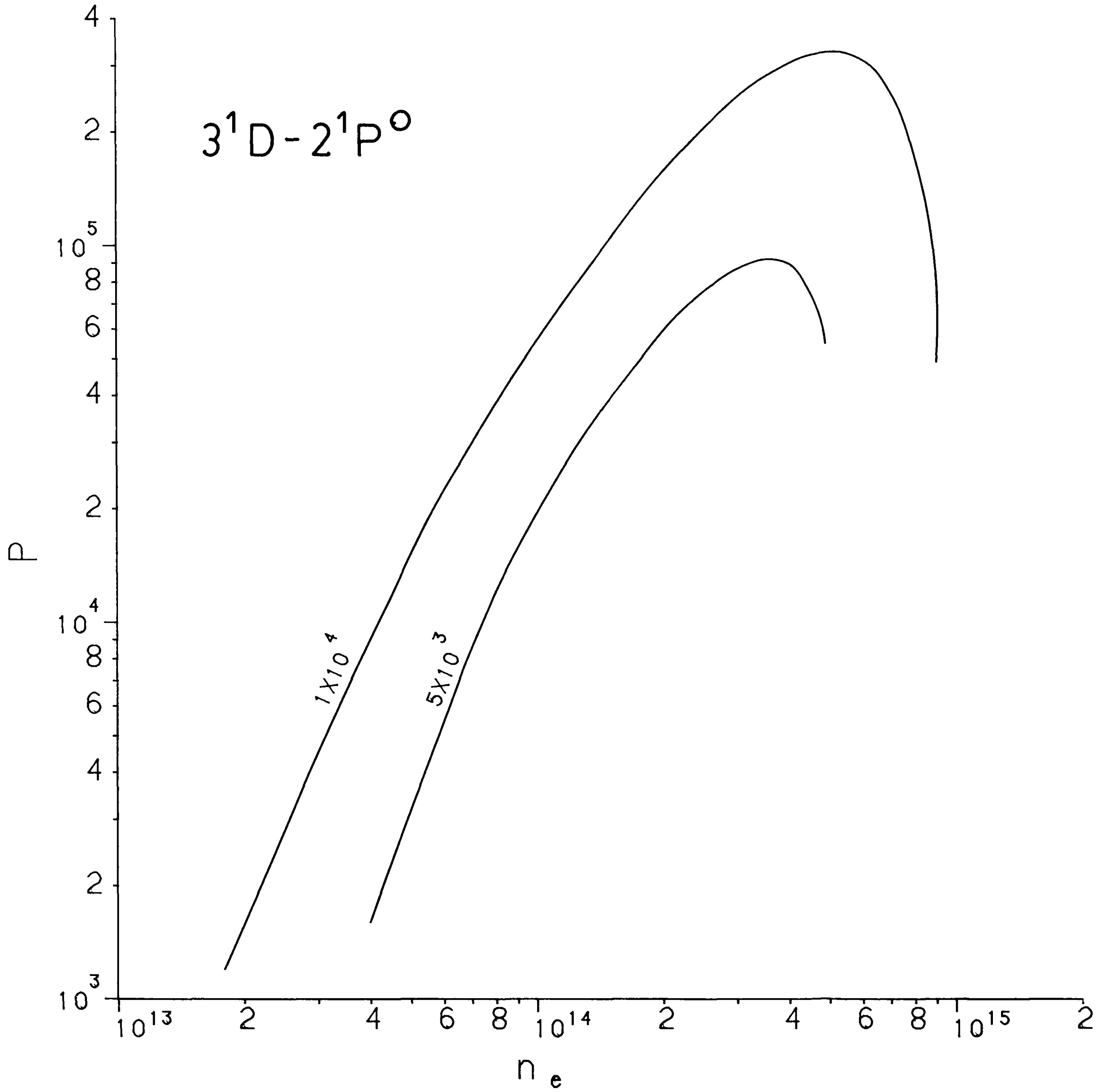

Fig. 3. P as a function of n_e for λ6678. The numbers by the side of curves represent T_e values in K.

gas (Gudzenko *et al.*, 1974, 1975; Otsuka, 1980). We then turn our attention to the observational evidence concerning these four lines in the spectra of Wolf–Rayet and allied objects.

Emission bands at 4922 and 6678 Å have been known for many years (Plaskett, 1924; Payne, 1933) in Wolf–Rayet spectra. Swings (1942) observed the spectra of several Wolf–Rayet stars; he lists the wavelengths for two of these. Swings and Jose (1950) observed the spectra of seven Wolf–Rayet stars in the range λλ6500–8800. Smith (1955) observed a large number of southern Wolf–Rayet stars, some in the range λ4000–λ6800, and some in λ3600–λ6800. He lists the wavelengths and intensities of lines for a few of these stars. The results of Swings (1942), Swings and Jose (1950), and Smith (1955) with respect to the four lines under consideration are summarized in Table II.

TABLE II

Relative intensities of the four lines in stars. A blank spot indicates that this region was not observed; 'abs.' stands for absent

Star	Relative intensity $\lambda 4922$	$\lambda 5048$	$\lambda 6678$	$\lambda 7281$
Swings (1942)				
HD 151932 (WN7)	abs.	abs.		
HD 164270 (WC8)	2	abs.		
Swings and Jose (1950)				
HD 184738 (WC8)		3	present	
HD 164270 (WC8)		4	2	
HD 168206 (WC7)		abs.	abs.	
HD 192103 (WC7)		3	1?	
HD 151932 (WN6+)		3		
HD 192163 (WN6)		8	2?	
HD 193077 (WN5)		2–3		
Smith (1955)				
HD 136488 (WC8)	2	abs.	6	
HD 119078 (WC7)	abs.	abs.	$W_\lambda = 20$	
HD 97152 (WC7)	abs.	abs.	present	
HD 115473 (WC6)	abs.	abs.	abs.	
HD 96548 (WN8)	$W_\lambda = 6.3$	$W_\lambda = 2.5$ [a]	$W_\lambda \sim 12$	
HD 92740 (WN7)	abs.	abs.	weak	
HD 143414 (WN6)	?	abs.	$W_\lambda = 25$ [b]	
HD 50896 (WN5+)	abs.	abs.	$W_\lambda = 21.2$	

[a] Blended with a N II line.

[b] Unusually strong as compared to other He I lines.

Andrillat (1957) carried out an extensive investigation of the spectra of WR stars in the red and near infrared regions. Her study covers the following categories of stars: four WC6 type, three WC7 type, one planetary nucleus of WC7 type, two planetary nuclei of WC8 type, eight WN5 type, six WN6 type, two WN7 type, and one WN8 type. He I $\lambda 6678$ is present in many of these, but $\lambda 7281$ appears to be absent.

For HD 192103, WC7, Underhill (1959) summarizes as follows: 'The first three members of the $2^1P - n^1D$ series, $\lambda\lambda 6678$, 4921, and 4388 appear in emission. There is some evidence for the leading member of the $2^1P - n^1S$ series, $\lambda 5047$, in emission...'. In HD 192163, WN6, Underhill (1959) found that $\lambda 6678$, $\lambda 4922$, and $\lambda 4388$ may be present weakly in emission.

For HD 191769, WN6, Underhill (1967) states: 'The leading member, $\lambda 6678$, blends with He II $\lambda 6683$; $\lambda 4922$ may be present but the present spectra are inadequate here.'

Cohen *et al.* (1975) have published the spectra of two WC9 stars in the range $\lambda\lambda 5650-7350$ and another two WC9 stars in the range $\lambda\lambda 5650-6770$. He I $\lambda 6678$ is of medium strength; $\lambda 7281$ appears to be quite weak.

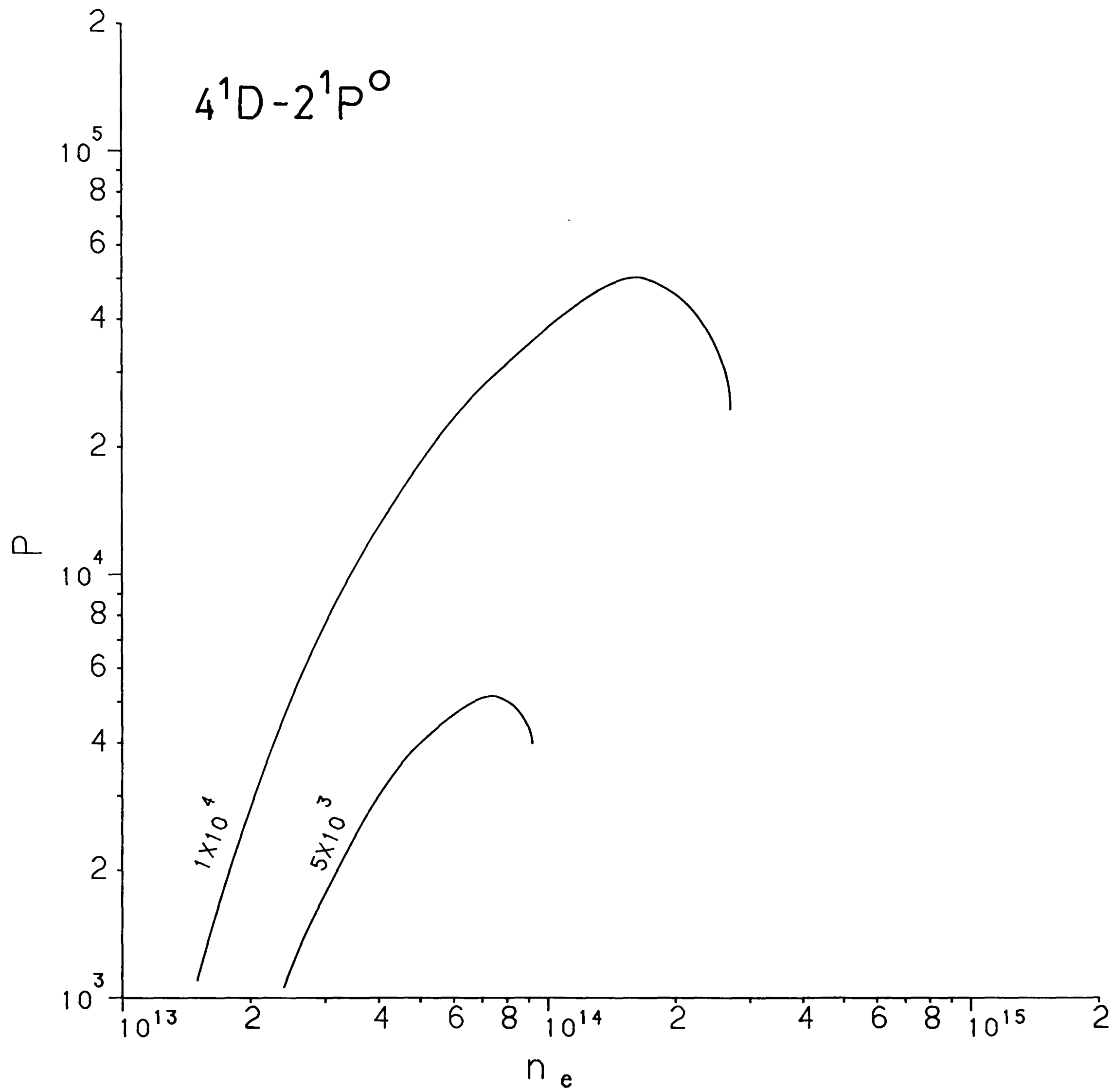

Fig. 4. P as a function of n_e for λ4921. The numbers by the side of curves represent T_e values in K.

Bromage and Nandy (1973) have obtained the spectrum of a WN5 star, Cyg OB2 No. WR2, in the wavelength range 5650 to 6850 Å. The micro-densitometer tracing shows a strong He I λ6678, with a width of 1350 km s^{-1}.

From the foregoing summary, we find that there are three stars, HD 143414, HD 50896, and Cyg OB2 WR2, which show unusually strong λ6678. The relative intensities of the four lines, that we are considering, amongst themselves and in relation to other He I lines in other stars (for which data are summarized above) do not seem to show a strong departure from the laboratory values and we shall not consider these stars further. There is one possible complication in the strength of the λ6678 line which we consider. He II λ6683 line lies close to it and it might be thought that it may contribute

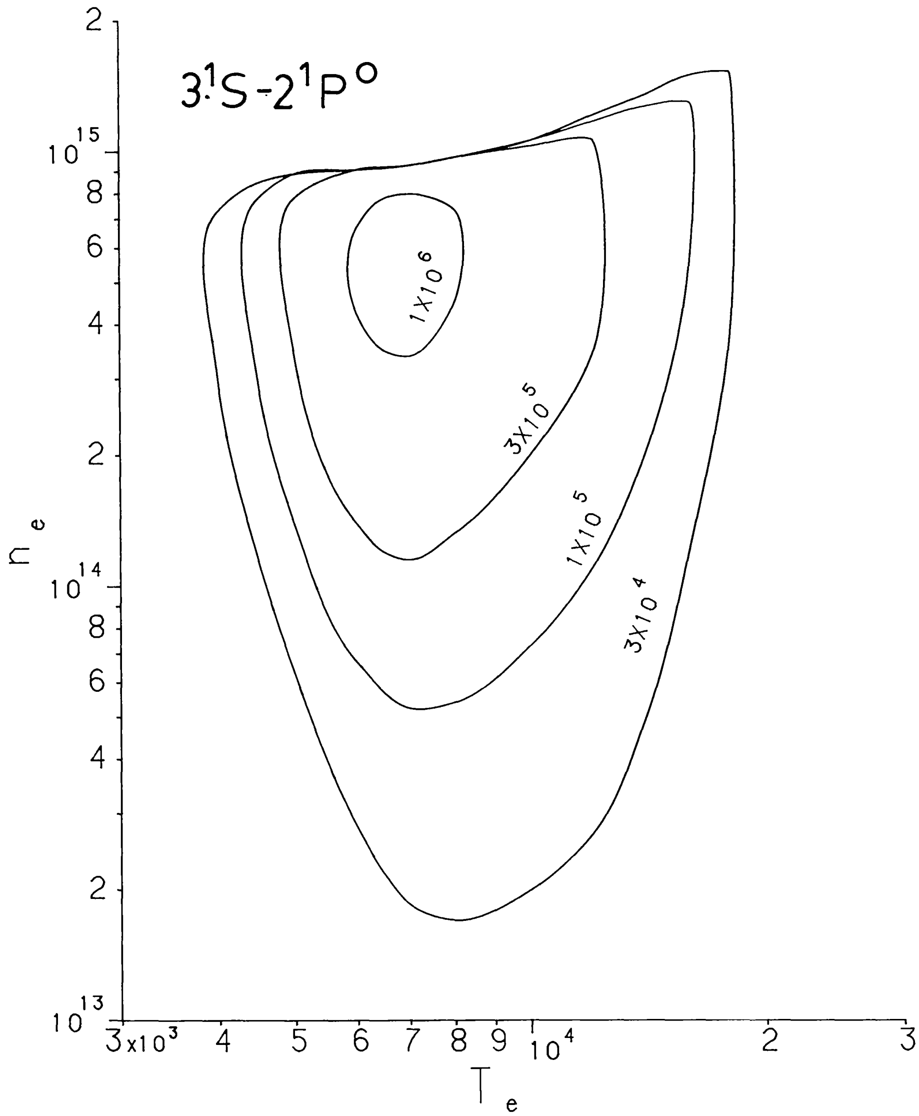

Fig. 5. P as a function of n_e and T_e for $\lambda 7281$. The numbers by the side of contours represent P values in cm^{-3}.

to the intensity of the observed line. The line He II $\lambda 6683$ arises from $n = 13$ to $n = 5$ and its A_{ki} value is $1.464 \times 10^5\ \mathrm{s}^{-1}$ (for comparison, $A_{ki} = 1.438 \times 10^8\ s^{-1}$ for $\lambda 4686$); thus this line is expected to be rather weak. Its contribution to the observed line can be estimated by looking at the intensity of other He II lines with comparable A_{ki} values. There is another line which arises from $n = 13$ and which falls in the visible region, He II $\lambda 4025.6$ ($n = 13$ to $n = 4$) and has $A_{ki} = 1.710 \times 10^5\ \mathrm{s}^{-1}$. Unfortunately, this line also falls at the position of a He I line, $\lambda 4026.2$. However, by looking at the intensity of other He I lines, the observed intensity of the $\lambda 4026$ line can be appropriately apportioned between He I $\lambda 4026.2$ and He II $\lambda 4025.6$. Thus, by scrutinizing the intensity of He I and He II lines, we find that the contribution of He II $\lambda 6683$ to the observed line in each of the three stars is quite small. We are, thus, led to conclude that very likely

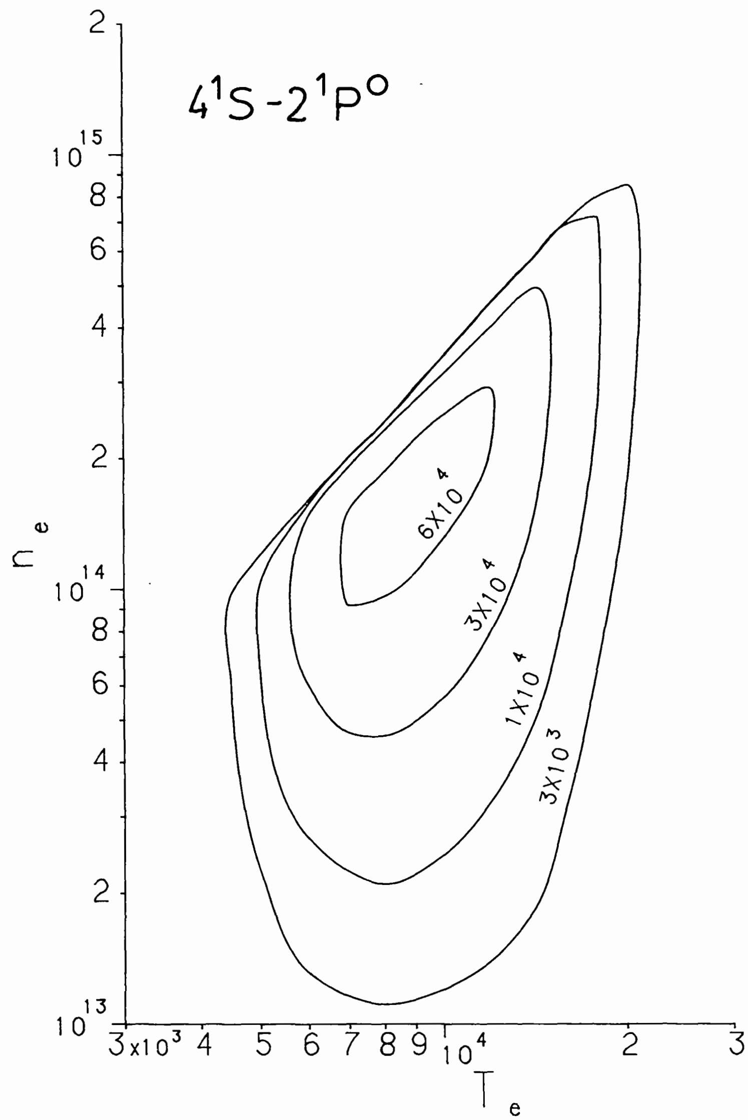

Fig. 6 $2P$ as a function of n_e and T_e for $\lambda5048$. The numbers by the side of contours represent P values in cm^{-3}.

laser action is responsible for the unusual strength of He I $\lambda6678$ in these three stars. The lines are broad which would indicate that the dominant mechanism for the population inversion is rapid cooling through expansion.

We note here that all the four lines, $\lambda\lambda4921$, 5048, 6678, and 7281 are known to occur in novae and nova-like stars with good strength (Andrillat, 1964; Meinel *et al.*, 1975). Next we consider two other interesting objects, the emission lines in which are relatively narrow.

The spectrum of the remarkable emission line star CPD$-56°8032$ in the red and infrared has been described by Thackeray (1977). His Table II shows a remarkable thing: He I $\lambda6678$ has an intensity of 9, while He I $\lambda7281$ has an intensity of 10. This indicates evidence of population inversion in the transition responsible for $\lambda7281$. For

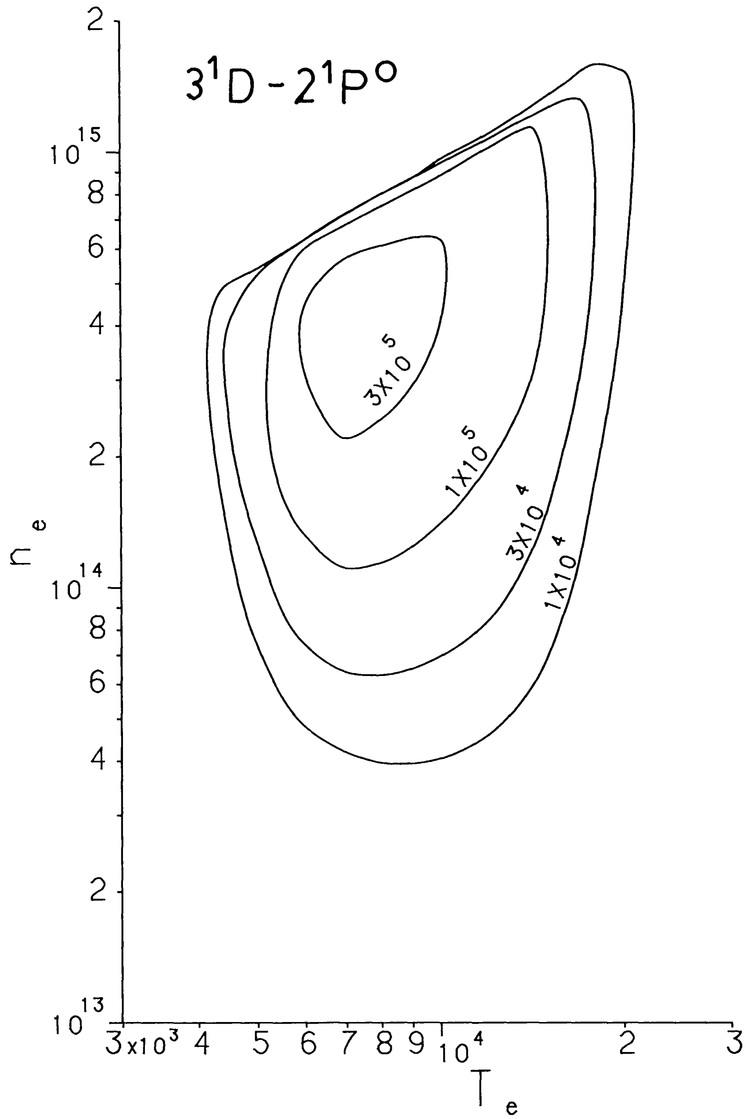

Fig. 7. P as a function of n_e and T_e for $\lambda 6678$. The numbers by the side of contours represent P values in cm^{-3}.

this reason we discuss this object in some detail. CPD−56°8032 was discovered by Bidelman *et al.* (1968). Cowley and Hiltner (1969) have described the spectrum of this star as unique. They have identified lines on their Cerro Tololo spectra from 3712 to 4714 Å. The spectrum contains a continuum, strong, slightly broadened C II emission lines and fainter emission lines from ions such as He I, C III, O II, O III, Si III, and Si IV, many of the latter group having P Cygni profiles. The blue hydrogen emission lines are sharp and extremely weak and [O II] is present. Webster and Glass (1974) have compared the object with M4–18, He 2–113, and V348 Sgr. Their discussion indicates that these four objects belong to a class, with low excitation surrounding nebulae, infrared radiation from dust grains and characteristic spectra dominated by C II emission lines. They appear to form a cool extension to the carbon sequence of

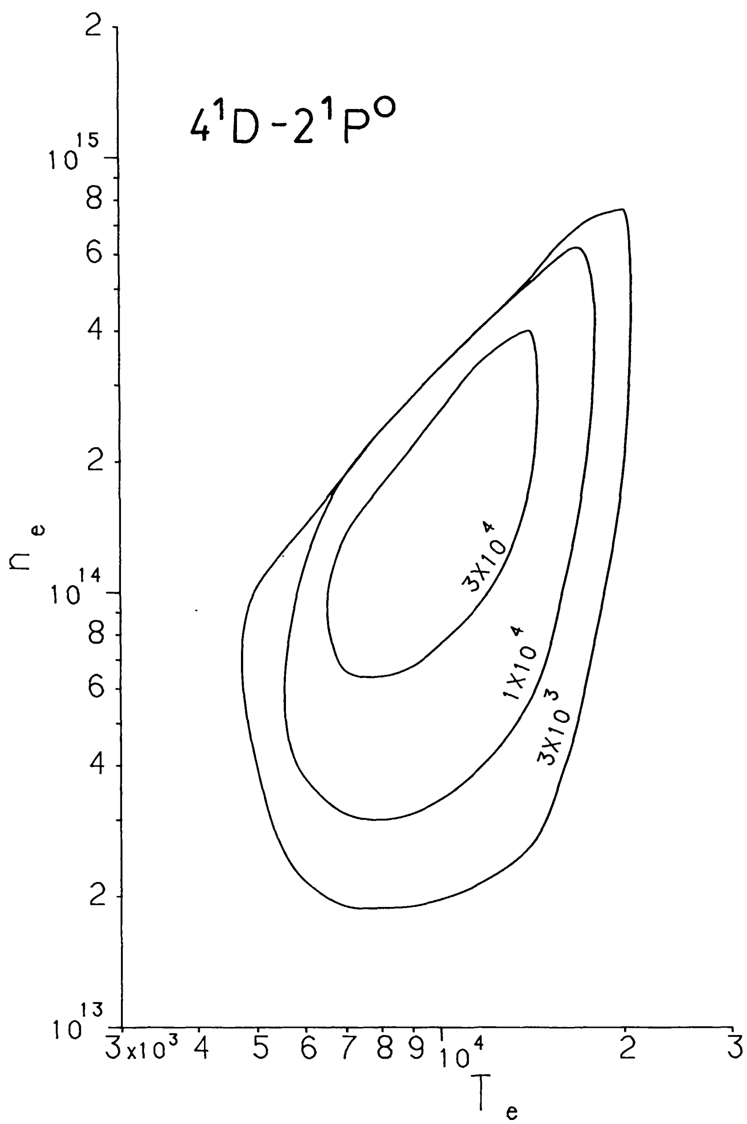

Fig. 8. P as a function of n_e and T_e for λ4921. The numbers by the side of contours represent P values in cm^{-3}.

Wolf–Rayet stars, the ionization level in the stellar atmosphere is lower than in stars classified as WC9. The emission lines in CPD−56°8032 are relatively narrow (~ 2 Å).

Downes (1984) has observed an emission line star, MWC84 = KPD 0415 + 5552, which has strong He I emission lines. The tracing shows λ6678 to be a strong line, much stronger than λ4921 (by a factor of 15 or so). Approximate visual estimates (made from the tracing) of the relative intensities of He I lines in this star are given in following in the parenthesis: λ4388 (0.2), λ4471 (0.5), λ4713 (0.5), λ4921 (0.5), λ5015 (1.5), λ5876 (10), λ6678 (10). The relative intensities of these lines in the laboratory are 3, 7, 4, 4, 6, 11, and 6, respectively (Moore, 1945). The relative strength of the line λ6678 is seen to be much stronger in this star than in the laboratory, and it appears highly likely that this is due to population inversion. Downes's tracing extends only up to about 7000 Å. It would be of interest to investigate this star at longer wavelengths and to examine the behaviour of $\lambda 7281$.

As mentioned earlier, the emission in these two objects are relatively narrow. The available evidence indicates that the expansion mechanism for cooling of the ejected plasma is not expected to be very effective, but rather it is the second mechanism referred to in the Introduction which appears to be responsible for the laser action in the $\lambda 7281$ line in CPD$-56°8032$ and that in the line $\lambda 6678$ in MWC84. Both experimental (Silfvast *et al.*, 1979; Dixon and Elton, 1977; Dixon *et al.*, 1978) and theoretical results (Wagli and Bohn, 1980) show that expansion of a plasma into an ambient gas can lead to population inversion.

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References

Ali, A. W. and Jones, W. W.: 1975, *Proc. Twelfth Internat. Conf. on Phenomena in Ionized Gases*, North-Holland Publ. Co., Amsterdam, p. 212.
Ali, A. W. and Jones, W. W.: 1976, *Phys. Letters* **55A**, 462.
Andrillat, Y.: 1957, *Suppl. Ann. Astrophys.* **2**.
Andrillat, Y.: 1964, *Ann. Astrophys.* **27**, 486.
Bates, D. R. and Damgaard, A.: 1949, *Phil. Trans. Roy. Soc. London* **A242**, 101.
Bates, D. R. and Kingston, A. E.: 1963, *Planetary Space Sci.* **11**, 1.
Bates, D. R., Kingston, A. E., and McWhirter, R. W. P.: 1962a, *Proc. Roy. Soc.* **A267**, 267.
Bates, D. R., Kingston, A. E., and McWhirter, R. W. P.: 1962b, *Proc. Roy. Soc.* **A270**, 155.
Bhagavatula, V. A. and Yaakobi, B.: 1978, *Opt. Commun.* **24**, 331.
Bidelman, W. P., MacConnell, D. J., and Bond, H. E.: 1968, *IAU Circ.*, No. 2089.
Bohn, W. L.: 1971, *Tenth Internat. Conf. on Phenomena in Ionized Gases*, Donald Parsons and Co., Oxford, p. 386.
Bohn, W. L.: 1974, *Applied Phys. Letters* **24**, 15.
Bohn, W. L.: 1978, *Bull. Am. Phys. Soc.* **23**, 152.
Boiko, V. A., Bryunetkin, B. A., Duvanov, B. N., Dyakin, V. M., Pikuz, S. A., Skobolev, I. Yu., Faenov, A. Ya., Fedosimov, A. I., and Shilov, K. A.: 1981, *Pis'ma Zh. Tekh. Fiz.* **7**, 665 (*Soviet Tech. Phys. Letters* **7**, 285).
Boiko, V. A., Derzhiev, V. I., Korneichuk, V. I., and Yakovlenko, S. I.: 1982, *Soviet Phys.-Lebedev Institute Rep.* **11**, 5.
Brocklehurst, M.: 1972, *Monthly Notices Roy. Astron. Soc.* **157**, 211.
Bromage, G. E. and Nandy, K.: 1973, *Nature* **241**, 30.
Burgess, A.: 1964, *Proceedings Symposium on Atomic Collision Processes in Plasmas*, Culham, UKAEA, Rep. No. 4818, p. 36.
Burgess, A. and Percival, I. C.: 1968, *Adv. Atom. Molec. Phys.* **4**, 109.
Burk, P. G., Cooper, J. W., and Ormonde, S.: 1969, *Phys. Rev.* **183**, 245.
Cohen, M., Barlow, M. J., and Kuhi, L. V.: 1975, *Astron. Astrophys.* **40**, 291.
Cowley, A. P. and Hiltner, W. A.: 1969, *Astron. Astrophys.* **3**, 372.
Dewhurst, R. J., Jacoby, D., Pert, G. J., and Ramsden, S. A.: 1976, *Phys. Rev. Letters* **37**, 1265.
Dixon, A. J., Harrison, M. F. A., and Smith, A. C. H.: 1976, *J. Phys.* **B9**, 2617.
Dixon, R. H. and Elton, R. C.: 1977, *Phys. Rev. Letters* **38**, 1072.
Dixon, R. H., Seeley, J. F., and Elton, R. C.: 1978, *Phys. Rev. Letters* **40**, 122.
Donaldson, F. G., Hender, M. A., and McConkey, J. W.: 1972, *J. Phys.* **B5**, 1192.
Downes, R. A.: 1984, *Publ. Astron. Soc. Pacific* **96**, 807.
Drawin, H. W.: 1963, *Atomic Cross Section for Inelastic Electronic Collisions*, Euratom-C.E.A. Report EUR-CEA-FC-236.
Drawin, H. W.: 1964, *Z. Naturfosch.* **19a**, 1451.

Drawin, H. W.: 1967, *Collision and Transport Cross Sections*, Euratom-C.E.A. Report EUR-CEA-FC-383.
Drawin, H. W.: 1969, *Z. Physik* **225**, 470.
Drawin, H. W., Emard, F., and Katsonis, K.: 1973, *Z. Naturforsch.* **28a**, 1422.
Filippov, S. S. and Yevstigneyev, V. V.: 1975, *Proc. Twelfth Internat. Conf. on Phenomena Ionized Gases*, North-Holland Publ. Co., Amsterdam, p. 41.
Flannery, M. R. and McCann, K. J.: 1975, *Phys. Rev.* **A12**, 846.
Flannery, M. R., Morrison, W. F., and Richmond, B. L.: 1974, *J. Appl. Phys.* **46**, 1186.
Fujimoto, T.: 1979, *J. Quant. Spectr. Rad. Trans.* **21**, 439.
Fujimoto, T., Sugiyama, I., and Fukuda, K.: 1972, *Mem. Faculty Eng. Kyoto Univ.* **34**, 249.
Furukane, U. and Oda, T.: 1984, *Z. Naturforsch.* **39a**, 132.
Furukane, U. and Oda, T.: 1985, *J. Quant. Spectr. Rad. Trans.* **33**, 645.
Furukane, U. and Yamamoto, T.: 1980, *Japanese J. Appl. Phys.* **19**, L285.
Furukane, U., Tsuji, Y., and Oda, T.: 1982, *J. Quant. Spectr. Rad. Trans.* **27**, 557.
Furukane, U., Yokota, T., Kawasaki, K., and Oda, T.: 1983, *J. Quant. Spectr. Rad. Trans.* **29**, 75.
Furukane, U., Yokota, T., and Oda, T.: 1979, *J. Quant. Spect. Rad. Trans.* **22**, 239.
Galushkin, Yu. I.: 1972, *Zh. Priklad. Spektrosk.* **16**, 788 (*J. Appl. Spectrosc.* **16**, 581).
Garstang, R. H.: 1967, *Astrophys. J.* **149**, 579.
Goldfarb, V. M., Ilina, E. V., Kostygova, I. E., and Lukyanov, G. A.: 1969, *Opt. Spektrosk.* **27**, 108.
Gordiets, B. F., Gudzenko, L. I., and Shelepin, L. A.: 1968, *J. Quant. Spectr. Rad. Trans.* **8**, 791.
Gordiets, B. F., Gudzenko, L. I., and Shelepin, L. A.: 1968a, *Zh. Priklad. Mekh. Tekh. Fiz.* **9**, 115 (*J. Appl. Mech. Tech. Phys.* **9**, 734).
Gordiets, B. F., Gudzenko, L. I., and Shelepin, L. A.: 1968b, *Zh. Eksperim. Teor. Fiz.* **55**, 942 (*Soviet Phys-JETP* **28**, 489).
Gudzenko, L. I., Filippov, S. S., and Shelepin, L. A.: 1966, *Zh. Eksperim. Teor. Fiz.* **51**, 1115 (*Soviet Phys-JETP* **24**, 745).
Gudzenko, L. I. and Shelepin, L. A.: 1963, *Zh. Eksperim. Teor. Fiz.* **45**, 1445.
Gudzenko, L. I. and Shelepin, L. A.: 1963, *Zh. Eksperim. Teor. Fiz.* **45**, 1445 (*Soviet Phys.-JETP* **18**, 998).
Gudzenko, L. I. and Shelepin, L. A.: 1965, *Dokl. Akad. Nauk SSSR* **160**, 1296 (*Soviet Phys.-Dokl.* **10**, 147).
Gudzenko, L. I., Mamachun, A. T., and Shelepin, L. A.: 1967, *Zh. Tekh. Fiz.* **37**, 598 (*Soviet Phys.-Tech. Phys.* **12**, 598).
Gudzenko, L. I., Shelepin, L. A., and Yakovlenko, S. I.: 1974, *Usp. Fiz. Nauk* **114**, 457 (*Soviet Phys.-Usp.* **17**, 848).
Gudzenko, L. I., Shelepin, L. A., and Yakovlenko, S. I.: 1975, in N. G. Basov (ed.), *Theoretical Problems in the Spectroscopy and Gas Dynamics of Lasers*, Consultants Bureau, New York, p. 103.
Hara, T., Kodera, K., Hamagaki, M., Matsunaga, K., Inutake, M., and Dote, T.: 1980a, *Japanese J. Appl. Phys.* **19**, L386.
Hara, T., Kodera, K., Hamagaki, M., Matsunaga, K., Inutake, M., and Dote, T.: 1980b, *Japanese J. Appl. Phys.* **19**, L606.
Hara, T., Ohgo, T., Hamagaki, M., and Dote, T.: 1982, *Japanese J. Appl. Phys.* **21**, L461.
Hara, T., Ohgo, T., Hamagaki, M., and Dote, T.: 1985, *Phys. Letters* **111A**, 285.
Hess, R. and Burrell, F.: 1979, *J. Quant. Spectr. Rad. Trans.* **21**, 23.
Hoffmann, P. and Bohn, W. L.: 1972, *Z. Naturforsch.* **27a**, 878.
House, L. L.: 1964, *Astrophys. J. Suppl.* **8**, 307.
Ilmas, M. and Nugis, T.: 1982, *Tartu Astrofuus Obs. Teated*, No. 67.
Irons, F. E. and Peacock, N. J.: 1974, *J. Phys.* **B7**, 1109.
Jacobs, V. L.: 1973, NASA TM X-70496.
Jacoby, D., Pert, G. J., Ramsden, S. A., Shorrock, L. D., and Tallents, G. J.: 1981, *Opt. Comm.* **37**, 193.
Jobe, J. D. and St. John, R. D.: 1967, *Phys. Rev.* **164**, 117.
Johnson, L. C.: 1967, *Phys. Rev.* **155**, 64.
Johnson, L. C. and Hinnov, E.: 1969, *Phys. Rev.* **187**, 143.
Jones, W. W. and Ali, A. W.: 1975, *Appl. Phys. Letters* **26**, 450.
Jones, W. W. and Ali, A. W.: 1977, *J. Appl. Phys.* **48**, 3118.
Jones, W. W. and Ali, A. W.: 1978, *J. Phys.* **B11**, L87.
Key, M. H., Lewis, C. L. S., and Lamb, M. J.: 1979, *Opt. Comm.* **28**, 331.
Khayarallah, G. A., Chen, S. T., and Rumble, J. R.: 1978, *Phys. Rev.* **17**, 513.
Kono, A. and Hattori, S.: 1984, *Phys. Rev.* **A29**, 2981.
Korukhov, V. V., Nikulin, N. G., and Troshin, B. I.: 1982, *Kvantovaya Elektron.* **9**, 1711 (*Soviet J. Quant. Electron.* **12**, 1099).
Limbaugh, C. C. and Mason, A. A.: 1971, *Phys. Rev.* **A4**, 2368.

Lin de Barros, H. G. P. and Brandi, H. S.: 1975, *Can. J. Phys.* **53**, 2289.
Luk'yanov, G. A.: 1977, *Zh. Tekh. Fiz.* **47**, 600 (*Soviet Phys. Tech. Phys.* **22**, 361).
Luk'yanov, G. A., Nazarov, V. V., and Pavlova, N. O.: 1978, *Opt. Spektrosk.* **44**, 47 (*Opt. Spectrosc.* **44**, 26).
Marr, G. V. and West, J. B.: 1976, *Atomic Data and Nuclear Data Tables* **18**, 497.
Martin, W. C.: 1973, *J. Phys. Chem. Ref. Data* **2**, 257.
Meinel, A. B., Avni, A. F., and Stockton, M. W.: 1975, 'Catalog of Emission Lines in Astrophysical Objects', University of Arizona Press, Tucson.
Millette, P. A. and Varshni, Y. P.: 1980, unpublished.
Miyake, S., Tsuchida, K., Uehara, Y., Sasaki, W., Watanabe, Y., and Fujita, J.: 1983, *Phys. Rev. Letters* **50**, 41.
Moiseiwitsch, B. L. and Smith, S. J.: 1968, *Rev. Mod. Phys.* **40**, 238.
Moore, C. E.: 1945, *A Multiplet Table of Astrophysical Interest*, NSRDS-NBS 40, U.S. Government Printing Office, Washington, D.C.
Moos, H. W. and Woodworth, J. R.: 1973, *Phys. Rev. Letters* **30**, 775.
Moustafa Moussa, H. R., de Heer, F. G., and Schutten, J.: 1969, *Physica* **40**, 517.
Oda, T. and Furukane, U.: 1983, *J. Quant. Spectr. Rad. Trans.* **29**, 553.
Otsuka, M.: 1978, IPPJ-350, Institute of Plasma Physics, Nagoya University.
Otsuka, M.: 1980, *Appl. Optics* **19**, 3904.
Payne, C. H.: 1933, *Z. Astrophys.* **7**, 1.
Pert, G. J.: 1976, *J. Phys.* **B9**, 3301.
Pert, G. J.: 1979, *J. Phys.* **B12**, 2067.
Pert, G. J. and Ramsden, S. A.: 1974, *Opt. Comm.* **11**, 270.
Pert, G. J. and Tallents, G. J.: 1981, *J. Phys.* **B14**, 1525.
Plaskett, J. S.: 1924, *Publ. Dominion Astrophys. Obs. Victoria* **2**, 287.
Pochat, A., Rozuel, D., and Peresse, J.: 1973, *J. Physique* **34**, 701.
Rapp, D. and Englander-Golden, P.: 1965, *J. Chem. Phys.* **56**, 1072.
Reshetnyak, S. Ã. and Shelepin, L. A.: 1975, *Zh. Tekh. Phys.* **44**, 1724 (*Soviet Phys. Tech. Phys.* **19**, 1076).
Rice, J. K., Truhlář, D. G., Cartwright, D. C., and Trajmar, S.: 1972, *Phys. Rev.* **A5**, 762.
Sato, K., Shiho, M., Hosokawa, M., Sugawara, H., Oda, T., and Sasaki, T.: 1977, *Phys. Rev. Letters* **39**, 1074.
Silfvast, W. T., Szeto, L. H., and Wood II, O. R.: 1979, *Opt. Letters* **4**, 271.
Skorupski, A. A. and Suckewer, S.: 1974a, *J. Phys.* **B11**, 1401.
Skorupski, A. A. and Suckewer, S.: 1974b, *Phys. Letters* **46A**, 473.
Smith, H. J.: 1955, Ph.D. Thesis, Harvard University, Cambridge.
Suckewer, S., Hawryluk, R. J., Okabayashi, M., and Schmidt, J. A.: 1976, *Appl. Phys. Letters* **29**, 537.
Summers, H. P.: 1977, *Monthly Notices Roy. Astron. Soc.* **178**, 101.
Swings, P.: 1942, *Astrophys. J.* **95**, 112.
Swings, P. and Jose, P. D.: 1950, *Astrophys. J.* **111**, 513.
Tallents, G. J.: 1977, *J. Phys.* **B10**, 1769.
Tallents, G. J.: 1978, *J. Phys.* **B11**, L157.
Thackeray, A. D.: 1977, *Observatory* **97**, 165.
Ton-That, D., Manson, S. T., and Flannery, M. R.: 1977, *J. Phys.* **B10**, 621.
Trebes, J. E., Krishnan, M., and Hirshfield, J. L.: 1981, *Bull. Am. Phys. Soc.* **26**, 907.
Underhill, A. B.: 1957, *Publ. Dominion Astrophys. Obs. Victoria* **9**, 209.
Underhill, A. B.: 1967, *Bull. Astron. Inst. Neth.* **19**, 173.
Van den Bos: 1969, *Physica* **42**, 245.
Van Dyck, R. S., Jr., Johnson, C. E., and Shugart, H. A.: 1970, *Phys. Rev. Letters* **25**, 1403.
Van Rann, A. F. J., De Jongh, J. P., Van Eck, J., and Heideman, H. G. M.: 1971, *Physica* **53**, 45.
Van Regemorter, H.: 1962, *Astrophys. J.* **136**, 906.
Varshni, Y. P. and Lam, C. S.: 1976, *Astrophys. Space Sci.* **45**, 87.
Vriens, L.: 1966, *Phys. Rev.* **141**, 88.
Wagli, P. and Bohn, W. L.: 1980, *J. Appl. Phys.* **51**, 3601.
Webster, B. L. and Glass, I. S.: 1974, *Monthly Notices Roy. Astron. Soc.* **166**, 491.
Westerveld, W. B., Heideman, H. G. M., and Van Eck, J.: 1979, *J. Phys.* **B12**, 115.
Willett, C. S.: 1974, *Gas Lasers: Population Inversion Mechanisms*, Pergamon Press, New York.
Zhizhan, Xu and Guangyu, Yin: 1983, *Sci. Sinica* **A26**, 657.
Zhinzhikov, G. M., Luk'yanov, G. A., Nazarov, V. V., and Pavlova, N. O.: 1978, *Zh. Tekh. Fiz.* **48**, 949 (*Soviet Phys. Tech. Phys.* **23**, 554).